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An empirical assessment of the impact of the light direction on the relief inversion effect in shaded relief maps: NNW is better than NW

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ABSTRACT

Relief inversion (or terrain reversal) effect is a well-known phenomenon in cartography that occurs when shadow is the main depth cue for three-dimensional shape perception. *Light direction* has been suggested as the main cause of this effect. However, the prevalence of relief inversion effect with regard to the changing light direction is currently not established, and there is little empirical evidence on this subject. This article systematically assesses the influence of light direction on the accuracy of landform perception in shaded relief maps (SRM). In a controlled experiment, 27 participants were asked to identify concave and convex landforms in 128 SRMs using a 5-point Likert scale where answers varied from *clearly a valley* to *clearly a ridge*. Eight different scenes were illuminated from 16 light directions to obtain the 128 SRMs. Our findings clearly demonstrate that incident light at 337.5° north-northwest (NNW) yields the highest accuracy and confidence ratings in landform identification among the investigated light directions; and leads to higher accuracy scores than at the 315° (NW) which is conventionally used in SRMs. Thus, we propose an update to this convention and recommend the light source to be placed at 337.5° when creating SRMs.

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Hill shading; hillshade; shaded relief map; relief inversion effect; terrain reversal effect; light-from-above-left preference; left bias; overhead illumination bias; shape-from-shading; global convexity bias

Introduction and related work

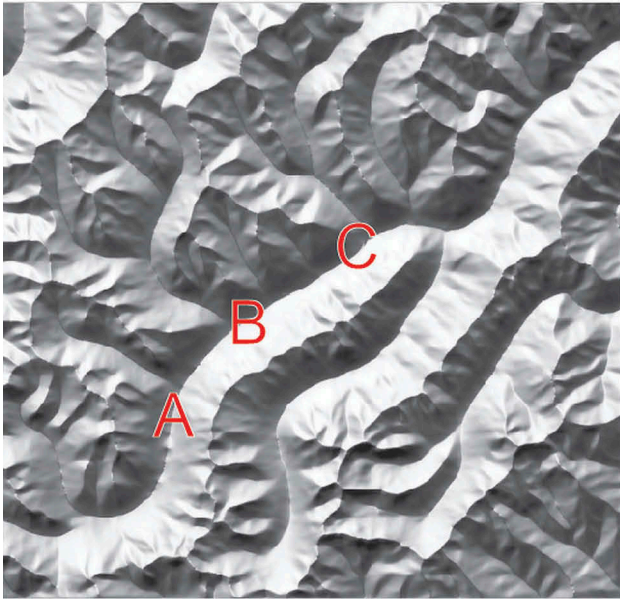
Cartographers are long aware of the phenomenon called *relief inversion effect*, also known as *terrain reversal effect*, which refers to an optical illusion in shaded relief maps (SRMs) as well as in satellite imagery where concave shapes (such as valleys) are perceived as convex (such as ridges) and *vice versa*, as demonstrated in Figure 1 (Imhof (1965) 2007; Saraf et al. 1996; Bernabé Poveda and Çöltekin 2014). Even though cartographers are well-aware of this phenomenon, and some ideas as well as solutions have been proposed to address it (Bernabé-Poveda, Sánchez-Ortega, and Çöltekin 2011; Gil, Arza, Ortiz and Avila 2014; Willett et al. 2015; Rudnicki 2000; Bernabé Poveda, Manso-Callejo, and Ballari 2005); research efforts appear to be limited toward understanding the underlying factors and establishing the prevalence of this perceptual fallacy in SRMs. In contrast, perceptual psychology has extensively investigated the visual system's extraction of three-dimensional (3D) shape from 2D shading in visual stimuli, broadly referred to as *shape from shading* in the related literature (e.g. Braun 1993; Ramachandran 1988; Langer and Bülhoff 2000; Liu and Todd 2004). In this article, we provide a brief overview of the perceptual psychology studies related

to relief inversion effect, link it to cartography, and present a controlled user experiment, in which we empirically study the prevalence (or absence) of the effect when the illumination direction is manipulated stepwise in equal intervals.

Factors that influence relief inversion effect

Several factors can influence the occurrence of relief inversion effect, e.g. the illumination direction, texture, shape discontinuity, and familiarity and/or previous knowledge (e.g. Howard 2002). We believe, in the case of terrain visualizations, familiarity may not play a very large role; since a valley and a ridge are both perfectly possible landforms and humans are familiar with both ridges and valleys. It is likely that we see mountains (albeit not ridges) more often than valleys; and it has been suggested that humans may have a global convexity preference, known as the *convexity bias* in perceptual psychology (Hill and Johnston 2007). Convexity bias suggests that in ambiguous convex/concave visualizations, viewers may opt for convex shapes (ridges) more often than concave shapes (valleys). Furthermore, those who work with terrain visualizations might be able to interpret landforms, e.g.

a) 337.5°



b) 157.5°

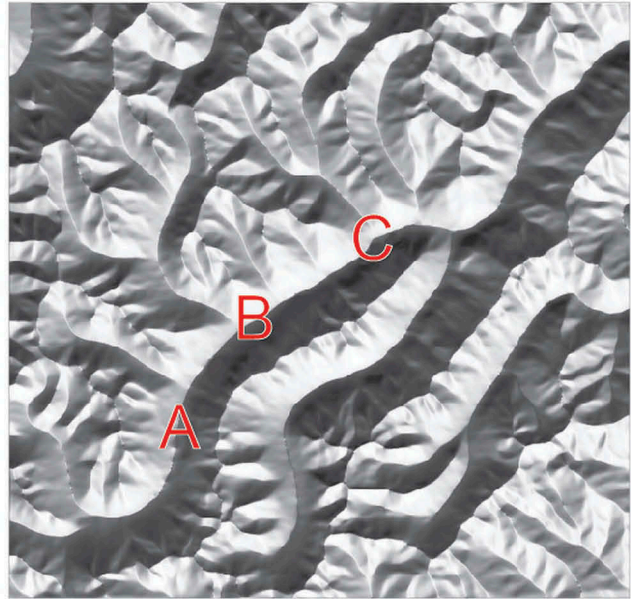


Figure 1. The same digital elevation model (DEM) is hillshaded under a) incident light from 337.5°, and b) from 157.5°. The marked landform (ABC) is a valley. Most observers perceive it correctly as a valley in the left (a), and inversely as a ridge in the right (b). The elevation angle of the light source is set to 45° in both images.

recognize river beds and similar cues. While familiarity might or might not help with solving the convex/concave ambiguity, illumination direction is possibly *very* relevant for terrain visualizations, and appears to be a dominant factor in most other cases as well. When there are no other depth cues to help with the shape discrimination, Kleffner and Ramachandran (1992) have demonstrated that the human visual system (HVS) assumes that the light shines from above (because natural illumination sources, such as the sun, is above us). This is known as the *overhead illumination bias* and it is a critically important theory for relief inversion effect, because this expectation of *light from above* determines the illusory perception of 3D shapes from the shading patterns. Interestingly, a *belief* that the light source is at a certain position can also affect the 3D shape perception experience (Berbaum, Bever, and Chung 1983). In other words, according to Berbaum, Bever, and Chung (1983), misinformation of the true light position can lead to wrong shape cognition. Similar evidence was also reported by Yonas, Kuskowski, and Sternfels (1979). Arguably, we do not observe this in the opposite situation, i.e. when a viewer is told the true light position in the presence of relief inversion effect; they do not seem to be necessarily able to override their perception. This can be tested with Figure 1: the exact position of the light source and the correct landform (valley) are provided

in the caption; does your perception change in Figure 1b because you know that this is a valley?

Another factor that may influence the relief inversion experience is the *background* of the viewer. It is well documented that there are individual and group differences between map users (e.g. Çöltekin, Fabrikant, and Lacayo 2010) and this is also suggested with regard to susceptibility to relief inversion effect in satellite images, i.e. experts and experienced map users may be interpreting the terrain differently (Bernabé Poveda & Çöltekin, 2014). Therefore, it is possible that factors such as map use frequency, being accustomed to northwest lighting in maps or broader knowledge of geomorphology may lead to a difference in accurate landform interpretation when the depth cues “give mixed signals.”

In a cartographic context, it has been suggested that relief inversion effect in SRMs with southern lighting can be alleviated to some degree by good map design, e.g. with rich shading and color (Imhof (1965) 2007). Specifically, Imhof suggested that overblending with hypsometric tints, accentuation of illuminated slopes with a yellow tone, addition of ground cover such as rivers or vegetation or emphasis of aerial perspective may help correcting the perceived landform (Imhof (1965) 2007; Patterson and Kelso 2004). Furthermore, certain characteristics (such as steepness, surrounding landform configuration) of a depicted landform may

influence the perception of relief inversion effect. According to Patterson (2016), relief inversion will take effect “when the light shines perpendicular to linear landforms,” “when linear landforms are situated amidst a plain,” or “when a canyon rather than a ridge is viewed” (last one probably, at least partially, because of the convexity bias).

Cartographic convention for shaded relief maps and perceptual psychology

In cartography, a map designer can essentially arbitrarily decide the position of the light source (most software allow the entire 360 degrees) to hillshade a digital elevation model (DEM). Nevertheless, most cartographers know and follow the cartographic convention of *upper-left (north-west) illumination*, i.e. 315° azimuth, 45° elevation (e.g. Slocum et al. 2008; Kraak and Ormeling 2010). It is expected that relief inversion effect will not occur at this light direction (Imhof (1965) 2007). This convention was suggested based on individual cartographers’ expert observations over the course of decades of experience, and it is (essentially) correct despite the fact that, in nature, sun is rarely in a north-west position in the northern hemisphere. This is because the convention is not necessarily determined by geography or topography, rather it is fundamentally linked to how the HVS functions. Several studies in perceptual psychology have attempted to measure the *overhead illumination bias* and observed that, if no other cues are present to indicate the illumination position, the HVS assumes the light to shine not exactly from overhead but from *slightly above left*, referred to as *light-from-above-left preference* or *left bias* (Gerardin, Kourtzi, and Mamassian 2010; Mamassian and Goutcher 2001; Sun and Perona 1998). The link between these observations in the perceptual psychology domain and the cartographic convention is not explicitly established in the literature until this point. However, we can assume that the evolution of the cartographic convention is an observation of the *left bias* in cartography domain. Furthermore, the light direction most often applied in art pieces “spanning two millennia” follow a similar convention; as they are typically illuminated by an applied light source at the above left (Sun and Perona 1998, p. 183). Sun and Perona (1998) also demonstrated that the preferred light direction was different for the left-handed and right-handed participants, even though both groups displayed a left bias (23.3° for right-handers and at 7.9° for left-handers). Independent from the perceptual psychology studies, cartographers also speculated that handedness may have played a role in light-from-above-left preference; thus implicitly, in the relief inversion effect (primarily by Imhof (1965) 2007). According to this thinking, majority of people (thus

majority of cartographers) are right-handed, thereby a preference for a (room or table) lighting from above-left has emerged to avoid obscuring writing and hand-drawn artwork. The application of the same light direction for hillshading would, therefore, be only logical. The implication here is that we eventually became too familiar with this light direction, and thus we may have a perceptual bias in favor of northwestern lighting (i.e. the left bias).

Research gap and contributions of this study

In this study, we empirically test the relationship between the illumination direction and the occurrence of relief inversion effect. This relationship, despite its wide acknowledgement, has not been studied systematically through a user experiment. Therefore, it is currently unknown which light directions actually lead to the best performance in 3D landform identification (i.e. the least amount of relief inversion effect); which light directions may lead to ambiguous judgments, and which light directions lead to the worst performance in 3D landform identification. This article fills this gap and empirically assesses the impact of 16 light directions (varied stepwise in equal intervals) on how viewers experience relief inversion effect. We conducted a rigorous controlled lab experiment and systematically measured the participants’ accuracy in landform identification using 128 terrain visualizations (eight original locations, each shaded under 16 light directions) with 27 participants. We also measured the confidence of the participants each time they made a decision. Specifically, we answer the following questions in this article: 1) Which light directions are most suitable for SRMs so that the occurrence of relief inversion effect remains minimal, and which light directions should be definitely avoided? 2) Do we empirically observe a *light-from-above-left preference (left bias)* when working with SRMs (and if yes, to what degree) as suggested in the perceptual psychology literature for nongeographic visual stimuli? In addition to these two main questions, we explore and report *group differences* in our observations and investigate if accuracy in landform identification and participants’ confidence in their own performance is affected by gender or experience.

The experiment

Experimental design: variables, stimuli and task

In a within-subject design, eight different DEMs (hillshaded with 16 different light directions varied stepwise in equal intervals) were used as the experimental stimuli (Figure 2). Note that throughout this manuscript, we use

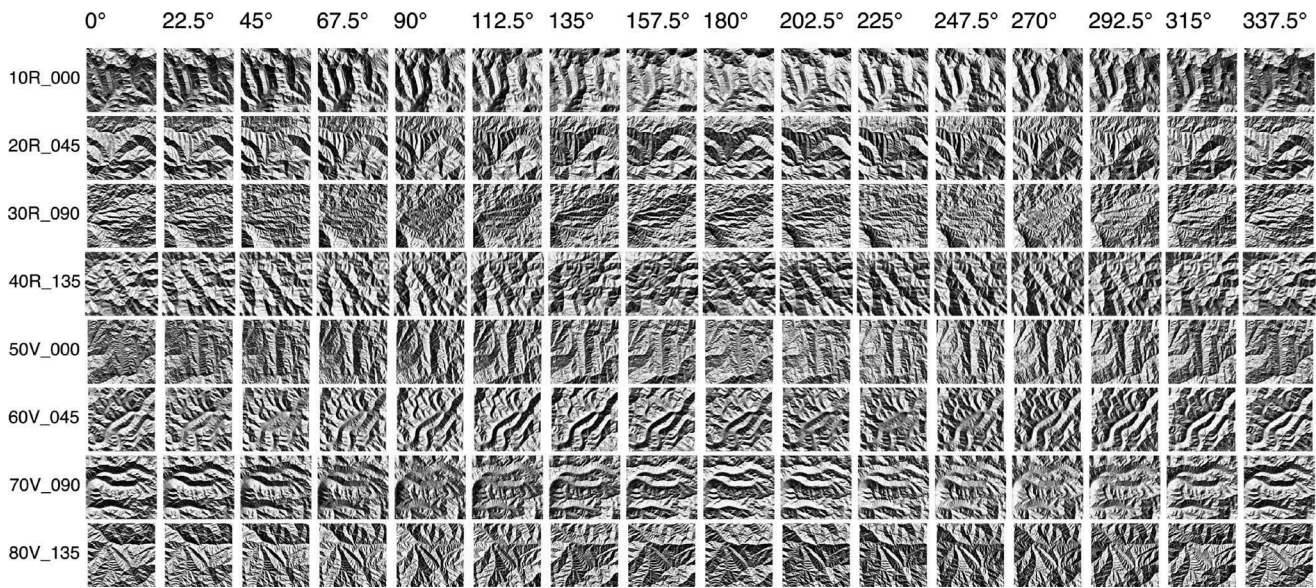


Figure 2. The 128 terrain visualizations that were used as experimental stimuli as thumbnail images (not to scale). Vertical axis shows the eight terrains, the horizontal axis shows the systematically changed light direction in equal steps (labels show the angles where the light source was placed).

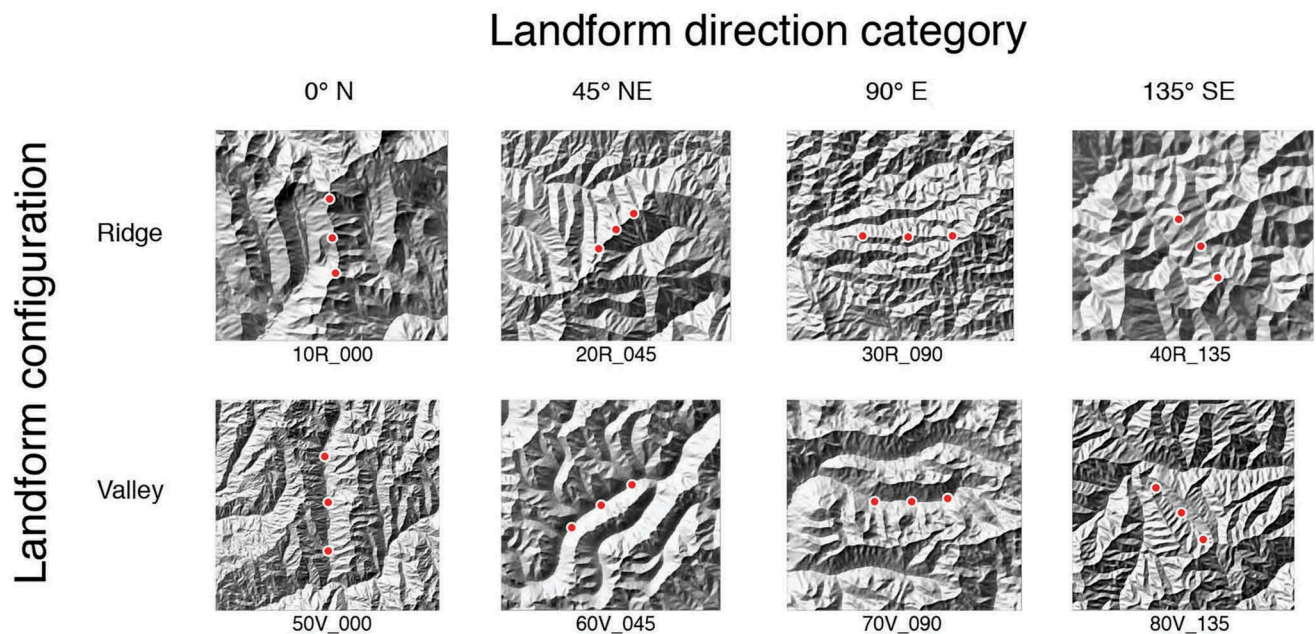


Figure 3. The eight terrains used in the experiment with their respective landform configuration and landform direction category. All images are lit from above-left. Note that for better visibility in small scale, the landforms in this figure are labeled with red dots instead of “ABC” as in the experiment.

the acronym DEM when we refer to the original eight digital elevation models, and the acronym SRM (shaded relief map) for the 128 hillshaded visualizations. *Light direction* was our main independent variable.

For control purposes, we selected terrains with linear features that systematically varied in *landform configuration* (four valleys, four ridges) as well as the

direction of the main features that we asked the participants to identify (Figure 3).

In each DEM (whose SRMs are shown in Figure 3), we selected a valley or a ridge feature that lies along a fairly straight line to control for similarity (thus comparability) between the studied stimuli. Four *landform direction categories* were used (0° N, 45° NE, 90° E, and 135° SE), thus

two out of the eight DEMs were (approximately) oriented to one of the four *landform direction categories*, respectively (“columns” in Figure 3). The naming of the terrains expresses these category memberships: the first two digits specify the terrain (instead of 1, 2, 3, we used 10, 20, 30...); *R* or *V* stand for *ridge* or *valley* and the last three digits indicate the given landform direction category. For example, 10R_000 is terrain number one, featuring a ridge which lies approximately at 0°; while 60V_045 is terrain number six, featuring a valley which lies approximately at 45°. This effort to counterbalance the landform types and directions is to avoid a potential bias that may be induced by the studied landform direction – light and shade relationships are important for what we are studying, and the orientation/direction of the linear landforms may play a role in how the shadows will be distributed over the display. More specifically, by varying landform directions, the effect of a possible confounding variable is minimized: When the landform direction is (nearly) parallel to the incident light, both slopes will have a similar shading tone, thus the studied valley or ridge will appear flat. This effect cannot be prevented, but through our design, it is distributed as evenly as possible over the whole angular spectrum. Similarly, we further ensured that both a valley and a ridge were represented in each landform direction (see each column in Figure 3). This way, a possible *global convexity bias* (e.g. Hill and Johnston 2007; Liu and Todd 2004; Langer and Bühlhoff 2001) is also distributed evenly among landform direction categories. Stimuli were presented in a random order to each participant to account for possible order effects such as learning or fatigue (the 128 stimuli were shown jointly with 99 additional displays for another – related, but separate – study, which is beyond the scope of this article). To further address the learning effect as well as possibly help with the fatigue effect; 15 memory distractors were added in random places in the form of humorous images.

As dependent variables, we monitored participants’ response *accuracy* in the given task (landform identification) and participants’ *confidence* in their judgment. There was a single task throughout the experiment. A landform was to be identified, which was always labeled with “ABC,” as in Figure 4, using the same color (i.e. red) and same order.

As shown in Figure 4 below the image (and above the caption); participants were asked to indicate whether they perceived the marked landform as a valley or a ridge on a 5-point Likert scale. If they were not sure, they could choose *ambiguous* or indicate their level of (un)certainty with the options 2 and 4. This way we also measured participants’ confidence in their own judgment.

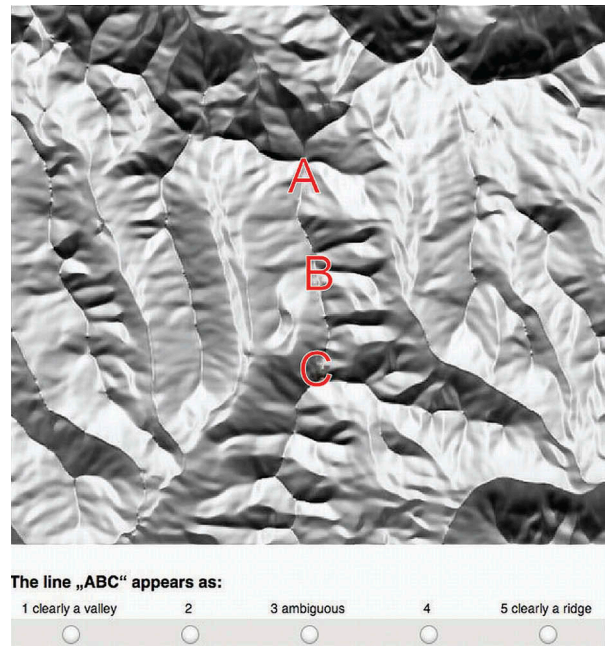


Figure 4. An example stimulus and the experimental task, as presented to the participants (not to scale).

Participants

We recruited 29 participants for this study. Two participants were excluded from the analyses because one participant seemingly misunderstood the task, and the other one marked nearly all landforms “ridge,” suggesting a very strong global convexity bias. Thus, data from 27 participants (14 females, 13 males; all right-handed except one) were used in analysis. All participants had normal or corrected-to-normal vision. One participant reported to have red-green color blindness. We kept this participant’s data, after checking that the colors (grayscale SRMs with red labels) did not hinder the participant. Participants’ age range was between 19 and 79 while majority of them (21 out of 27) were between 19 and 39, five of them 60–79, and one older than 70, and the average age was 35.1 years. Seven participants were high school graduates while 20 participants had a bachelor’s degree or higher. None of the participants reported to have used SRMs “daily” while 16 of them marked “never/rarely” and 11 “occasionally/often.” Similarly, 17 participants reported “little or no experience” in cartography/GIS, while 10 reported “professional or nearly professional experience.” The participants were given a voucher (worth of CHF 5) for the local cafeteria and offered a small bar of chocolate to take away at the end of the experiment as a reward.

Materials: preparation of the stimuli

The DEMs were extracted from ASTER GDEM V2 (<http://asterweb.jpl.nasa.gov/gdem.asp>) which was downloaded from the web service provided by United States Geological Survey (USGS) at <http://earthexplorer.usgs.gov>. Data is provided at a resolution of 1 arc-second, corresponding to approximately 30 m at the equator.

Table 1 shows the exact locations of the eight terrains (position of middle dot in Figure 3), their altitude above the sea level, altitude difference between marked landform and adjacent valley ground or ridge and width from West to East. The rough ASTER GDEMs were each smoothed with identical settings using the Terrain Sculptor software (Jenny 2010–2016; Leonowicz, Jenny, and Hurni 2010). The DEMs were then rendered with Landserf 2.3 (Wood 2016–2009) which uses Lambertian reflectance. The illumination elevation angle was always set to 45° as is commonly used in cartography for steep terrains. The rendering parameter *vertical exaggeration* was set to 0.7 and *aspect bias* to 50% for all DEMs. *Vertical exaggeration* controls the degree of shadow throughout the whole image and *aspect bias* determines the degree of influence of aspect rather than slope steepness to control the amount of shadow (Wood, 2009). These settings allowed achieving a good contrast between sun facing and averted slopes and overall pleasing results (subjectively assessed by the authors). Some visual examples can be seen in Figures 1–4.

Technical setup

The experiment was conducted on a Windows workstation (Dalco Intel Core i5 760) in a controlled lab environment at the eye movement laboratory at the Department of Geography of University of Zurich. The computer and internet speed, screen size, room lighting and temperature, and other environmental factors were kept constant. Stimuli were displayed through a browser on a 23-inch flat screen at a 1920 × 1080 screen resolution. Eye movements were recorded with a Tobii TX300 eye tracker at a 300 Hz sampling resolution; however, eye movement data

were collected primarily for the parallel study and were not analyzed for this publication.

Procedure

After welcoming and initial orientation, participants signed a consent form that provided general information about the experiment, and filled in a questionnaire about their backgrounds (age, gender, visual abilities, map use frequency, and their expertise in geography-related fields). Participants could choose either English or German for interacting with the experiment leader, or when working with the written tasks. For the main experiment, the participants were seated comfortably at the computer, the task (as in Figure 4) was introduced and verbally explained. To ease performance anxiety, they were informed that there is no wrong or right answer and that the study tests the images and not them. Furthermore, they were told that there were 242 visualizations in total (together with the 99 images that will be analyzed separately, and the 15 distractors) and that the experiment was expected to take about 15–20 min. At the beginning of the experiment, the participants were asked whether the task was clear and they were informed that they could ask clarification questions to the experimenter throughout the experiment if needed. To prevent them from overinterpreting or using other, external knowledge, they were also told to answer as quickly as they can, based on their perceptual experience. After the task instructions and the calibration for the eye movement recording, the main experiment started. Following the main experiment, participants responded to another questionnaire, where, among other things, relevant to the parallel study, they reported if their perception switched between convex and concave shapes during a task (i.e. if they saw a valley, then a ridge, then perhaps again a valley; we named this phenomenon as “terrain flipping”). Note that the participants were kept naïve during the experiment, and only were informed at the end that the experiment was about reversed terrain perception. Finally, they were asked if they had any further input or comments; were debriefed and thanked, their rewards were offered and the session thus ended.

Table 1. Terrain specifications.

Terrain	Latitude (°)	Longitude (°)	Altitude (m)	Altitude difference (m)	DEM width (km)
10R_000	−33.57	−69.87	4720	1000	15.9
20R_045	42.75	45.11	3740	1630	22.7
30R_090	42.23	83.48	3220	280	15.3
40R_135	−41.42	173.46	1130	820	15.2
50V_000	−33.36	−70.13	2240	1720	38.0
60V_045	−45.25	168.50	590	880	19.3
70V_090	49.10	86.97	2120	860	17.0
80V_135	37.31	75.75	3750	510	15.3

Results

Accuracy

The descriptive statistics immediately, and very clearly, revealed that the northern light directions lead to mostly correct landform identifications, whereas southern light directions lead to more occurrences of inverse perception; and when the light source is at the east or west, there is considerable ambiguity (Figure 5).

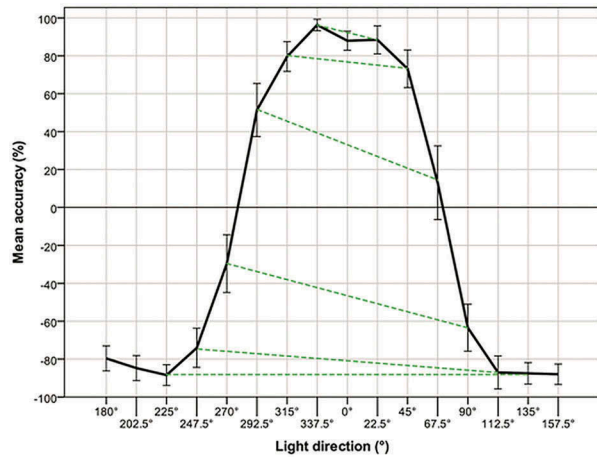


Figure 5. Overall success rates. The vertical axis is organized based on a “zero-line” where success and failure is equal. Percentage of success is marked towards 100% and failure towards -100%. The horizontal axis shows the angle of the light source. The error bars indicate 95% confidence intervals over all eight terrains. The green (dashed) lines link corresponding western and eastern light directions of the same angular inclination from 0°.

Figure 5 shows the overall *accuracy* (success rates) in landform identification under tested conditions.

Figure 5 reveals that light directions between 315° and 45° yield a fairly high accuracy (above 80%, also see Figure 6). The 337.5° yields the highest accuracy (96.3%) among all studied angles (inferential statistics comparing 337.5° to its immediate neighbors are provided later in the subsection “Evaluating the Cartographic Convention”). At 278.2° and 68.5° light directions (angles calculated as an interpolation), equal number of participants gave opposite answers, i.e. the number of correct answers is identical to the number of “inverse” answers (interception with the “zero-line” in Figure 5). Note that at this stage of the analysis, all

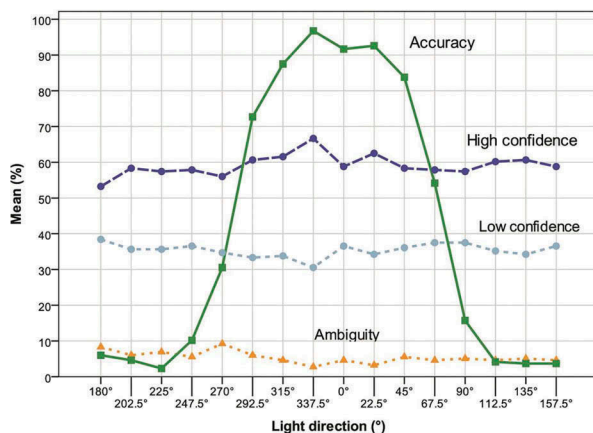


Figure 6. Overall mean accuracy, mean confidence, and mean ambiguity rates for all light directions.

27 participants were treated as one entity, i.e. we aggregated their responses for each light direction. The accuracy values were calculated as follows: The proportion of correct judgments (i.e. number of times participants responded “4” and “5” combined and divided by the total number of responses), *minus* the proportion of false judgments (i.e. number of times participants responded “1” and “2” combined and divided by the total number of responses). This means if a landform was deemed as a valley, e.g. 40% of the time, and ridge 40% of the time, we would get a zero, thus it would be on the zero line in Figure 5. When a difference of proportions is calculated, we get a clear picture of where there are more correct responses (0 to +100%), where participants have equal number of opposite answers (zero-line), and where there is more relief inversion effect, thus incorrect (or inverse) identification of the landforms (0 to -100%). In this case, the ambiguous rating (“3”) is neutralized, thus does not affect the accuracy calculations.

Furthermore, studying these accuracy results, we observed a *left bias*. Comparing corresponding western and eastern light directions of the same angular inclination from 0° (green dashed lines in Figure 5), we note that the participants perform better with western light directions for most angular pairs except for the pairs 225°/135° and 202.5°/157.5°. To measure whether these differences in accuracy are statistically significant, we compared each of the pairs with a McNemar test. McNemar test is a nonparametric test for repeated measures of two related dichotomous variables, which is appropriate for repeated measures designs when comparing paired proportions (McNemar 1947), such as in our study. The difference is significant for the lateral light direction pairs: 292.5° vs. 67.5°: $p = .000$ (1-tailed) < 0.01 ; 270° vs. 90°: $p = .000$ (1-tailed) < 0.01 and 247.5° vs. 112.5°: $p = .018$ (1-tailed) < 0.05 , as well as for 337.5° vs. 22.5°: $p = .002$ (1-tailed) < 0.01 . There is no indication of a *left bias* for the southern-most angular pairs: 225° vs. 135°: $p = .113$ (1-tailed) > 0.05 and 202.5° vs. 157.5°: $p = .304$ (1-tailed) > 0.05 (presumably because both sides have very low accuracy rates), as well as for 315° vs. 45°: $p = .202$ (1-tailed) > 0.05 . We further calculated the *left bias* in degrees from the 0°: We applied a smoothing function to fit a smooth line to the mean values of all participants’ accuracy ratings (calculation as of Figure 5). For the method, we used a linear model with the formula $y \sim ns(x, 7)$ with seven degrees of freedom. This number led to the most exact smooth line (yet assuring the smooth line to receive only one maximum). We received the curve maximum at 13.4° to the left from the vertical. This value thus denotes the theoretical light direction at which we would expect the highest accuracy rating for our given terrains.

Confidence

While the “3” ratings are ambiguous in terms of confidence (“I am not sure”); we interpreted the two extremes (“1” clearly a valley, and “5” clearly a ridge) as signs of *high confidence* and the remaining two ratings (“2” and “4”) as indications of *low confidence*. Figure 6 shows a combined plot of accuracy and confidence rates, where *accuracy* is the inverted U-curve; and *confidence* is the dashed lines. For Figure 6, the *accuracy* was calculated slightly differently than in Figure 5 (i.e. not as a difference): *Accuracy*, in this case, is the percentage of times a landform was perceived correctly (irrespective of low or high confidence). Note that in this calculation, we considered “ambiguous” ratings incorrect, as we expect that if a participant cannot tell a valley from a ridge, the visualization fails to facilitate its purpose.

What we see in Figure 6 is that participants are highly confident most of the time (approx. 60%) that what they were seeing was *clearly a valley* or *clearly a ridge*. We see considerably fewer low confidence cases (approx. 35%), and dramatically fewer cases of explicit ambiguity (approx. 5%). While accuracy is very high for northern and very low for southern light directions, participants’ confidence stays approximately constant for all light directions. This suggests that the inversion effect is very strong, as the participants are not aware whether their answers are correct or not. Nonetheless, interestingly, the *confidence* peaks at the light direction that *also* leads to most accurate results (337.5°) with 96.76% accuracy and 66.67% (high) confidence.

Evaluating the cartographic convention

To evaluate how our results compare to the current understanding of the cartographic best practice; we conducted further statistical analysis on the difference between 337.5° which is our “best performing” light direction (96.3% mean accuracy, SD: 4.9), and 315°, which is the cartographic convention (79.7% mean accuracy, SD: 23.6). Additionally, our descriptive

statistics revealed that the northern light direction (0°) also had more correct answers (88% mean accuracy rate, SD: 11.5) than the cartographic convention, suggesting that the more northern 337.5°–0° window may be safer than 315°–337.5° window. Therefore, in this section, we compare the 315° (NW) with 337.5° (NNW) as well as with the 0° (N) in a pairwise manner using inferential statistics. Figure 7 shows the mean accuracy ratings specifically for these light directions. To understand whether the difference between these accuracy rates (as shown in Figure 7) were statistically significant, we compared them with a pairwise McNemar test (Table 2). As can be seen in Table 2, participants’ judgments were labeled as *correct* or *inverse* (irrespective of confidence). Ambiguous judgments (i.e. “3”) were disregarded in the analysis, primarily because we are interested in the dichotomous accurate/inaccurate (correct/inverse) comparisons, and also because the number of people who marked “ambiguous” were rather small (for NNW 2.8%, for N

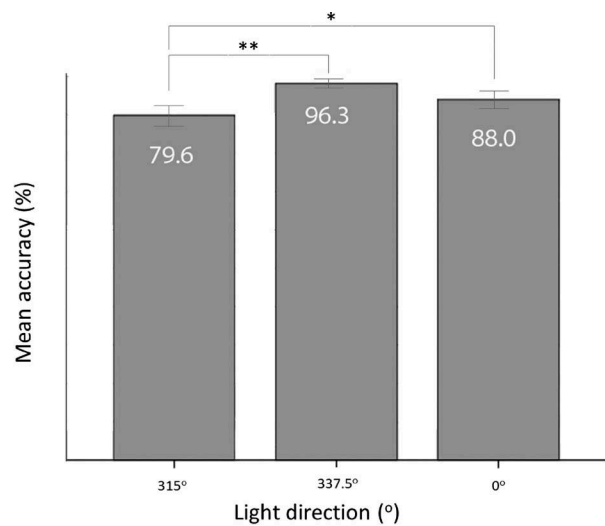


Figure 7. Accuracy in landform identification for the conventionally recommended light direction (315°) compared with 337.5° and 0°. Values show the exact mean accuracy ratings over all 27 participants’ accuracy rates (* = $p < .05$, ** = $p < .01$). The error bars indicate 95% confidence intervals.

Table 2. Changes in accuracy between light directions: comparison between a) 315° and 337.5°, b) 315° and 0°, and c) McNemar results for both. Both comparisons give significant results at 99% and 95% confidence, respectively.

a) 315° and 337.5°			b) 315° and 0°		
315°	337.5°		315°	0°	
	Inverse	Correct		Inverse	Correct
Inverse	1	16	Inverse	2	13
Correct	0	184	Correct	4	178
c) Test statistics ^a					
315° and 337.5°			315° and 0°		
N	201		197		
Exact sig. (1-tailed)	.000		.025		
Point probability	.000		.018		

a. McNemar test

and NW 4.6%). In total, we took 216 measurements (8 terrains * 27 participants) for each studied light direction. Results show that NW yields a statistically significantly lower accuracy than NNW ($p = .000$ (1-tailed) $<.01$), as well as N ($p = .025$ (1-tailed) $<.05$).

Exploratory analysis of group differences

In this section we offer an *exploratory* analysis of group differences based on self-reported experience (i.e. “frequency of SRM use”) and gender, and whether gender and experience have an impact on participants’ *accuracy* and *confidence* with the experimental tasks. Since we did not counterbalance the study specifically for group differences, these analyses are mainly to identify ideas for further testing, i.e. hypothesis building.

Experience

We expected that frequent SRM users would perform better than infrequent users because of their familiarity with the look of an SRM, and through their possible knowledge of geomorphology. In other words, they would automatically interpret more, and possibly respond differently even if they might not consciously note that southern lighting makes the terrain forms look “strange.” When asked to rate their frequency of use of SRMs on a 5-point Likert scale; of the 27 participants, seven indicated “never” (25.9%), nine “rarely” (33.3%), eight “occasionally” (29.6%), three “often” (11.1%), and none “daily.” We grouped the participants into two categories for the statistical analysis, and for convenience, we named the groups as *no experience* (“never/rarely”) and *some experience* (“occasionally/often”).

Descriptive statistics on *accuracy scores* suggested that participants with *some experience* have performed slightly better than those with *no experience* for most light directions. Interestingly, for very northern angles (337.5°, 0°, and 22.5°), participants with *some experience* perform slightly worse than participants with *no experience*, suggesting a possibility of overinterpretation (Figure 8). However, a Mann-Whitney U test yielded that the *overall* difference in accuracy scores (combined for all light directions) based on experience were not statistically significant for the two groups ($U = 21521$, $p = .43$).

Descriptive statistics regarding the *confidence ratings* also point at a potentially interesting difference between *no experience* and *some experience* groups (Figure 9). Figure 9 indicates that participants with *no experience* displayed more confidence than the participants with *some experience* for most light directions – with four exceptions; in which the ratings were close

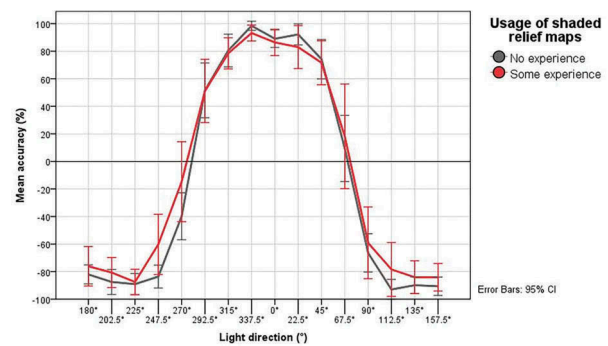


Figure 8. Accuracy of landform identification based on frequency of SRM use. The error bars indicate 95% confidence intervals.

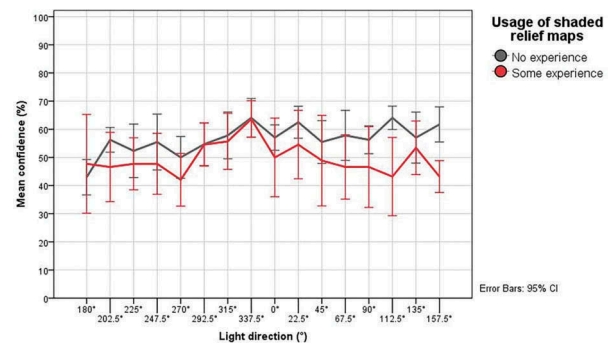


Figure 9. Participants’ confidence in their success based on their frequency of SRM use. The error bars indicate 95% confidence intervals.

(180°, 292.5°, 315°, and 337.5°). Overall (combined for all light directions), participants with *no experience* indicated higher confidence than those with *some experience* (Mann-Whitney $U = 25044$, $p = .046$).

Note that the *confidence rating* was calculated (unlike as in Figure 6) as the proportion of *high confidence* (i.e. number of times participants responded “1” and “5” divided by the total number of responses) minus the proportion of *ambiguous* judgments (i.e. number of times participants responded “3” divided by the total number of responses) in this section. An interesting pattern for confidence ratings is that for the northern light directions where the illusion is weakest and the two participant groups have similar accuracy rates, they also feel similarly confident (Figure 9). However, for the light directions that are more likely to lead to relief inversion effect, participants with *some experience* appear to “register something,” i.e. they may be experiencing more contradictions between their perception and their understanding of the terrain. To confirm this, however, a dedicated experiment that is designed to measure such differences (and control for other possible moderating factors) needs to be conducted.

Gender

Similarly, we explored if there may be a *gender difference*. We expected to see no differences in terms of performance (i.e. accuracy); however, it has been previously shown that confidence levels might be different for men and women in many tasks, such that men express higher confidence irrespective of their performance (e.g. Bengtsson, Persson, and Willenhag 2005; Wilkening and Fabrikant 2011). As expected, we did not observe a difference in overall *accuracy* in the experimental tasks between the two gender groups (Mann-Whitney $U = 22929$, $p = .78$); while the descriptive statistics suggested that on average, male participants demonstrated higher *confidence* for all light directions except in three cases (0° , 22.5° , and 157.5°). A Mann-Whitney U test, however, did not yield a statistically significant difference for confidence differences based on gender ($U = 20,940$, $p = .06$).

Interactions between gender and experience

As mentioned, we observed that *no experience* group indicated overall higher confidence than *some experience* group. On the other hand, even though there are considerably more females with *no experience* (11 women, 5 men) than *some experience* (8 men, 3 women), descriptive statistics suggested that the male participants rated their confidence somewhat higher on average. Therefore, we studied the interactions between gender and experience. A breakdown of how gender and experience interact with accuracy and confidence revealed an interesting pattern (Figure 10).

In this study, men with no experience marked the highest levels of confidence, where the remaining groups demonstrated similar levels of confidence (Kruskal-Wallis $H = 25.11$, $df = 10$, $p = .005$, Figure 10, right), irrespective of their accuracy i.e. the most confident group was not any more accurate than the other groups (Kruskal-Wallis $H = 6.75$, $df = 16$, $p = .98$, Figure 10, left). In other words, *men with some experience*, *women with some experience*, and *women with no experience* demonstrate similar levels of confidence, while *men with no experience* demonstrate clear overconfidence – irrespective of their accuracy scores. Our earlier observation that maybe more experienced users “register something” might be indeed the case, however, these results suggest that gender appears to be a moderating factor (although, the usual disclaimer applies: there may also be other factors that are not controlled in this study, thus for a generalizable argument, we need further studies).

Discussion

Accuracy of landform identification: what is the best light direction?

In this study we empirically investigated how the light direction impacts the prevalence of relief inversion effect in SRMs. Our findings clearly demonstrated that, overall, incident light *from above* (i.e. north) leads to most correct identification of 3D landforms with the SRMs, whereas light from “below” (i.e. south)

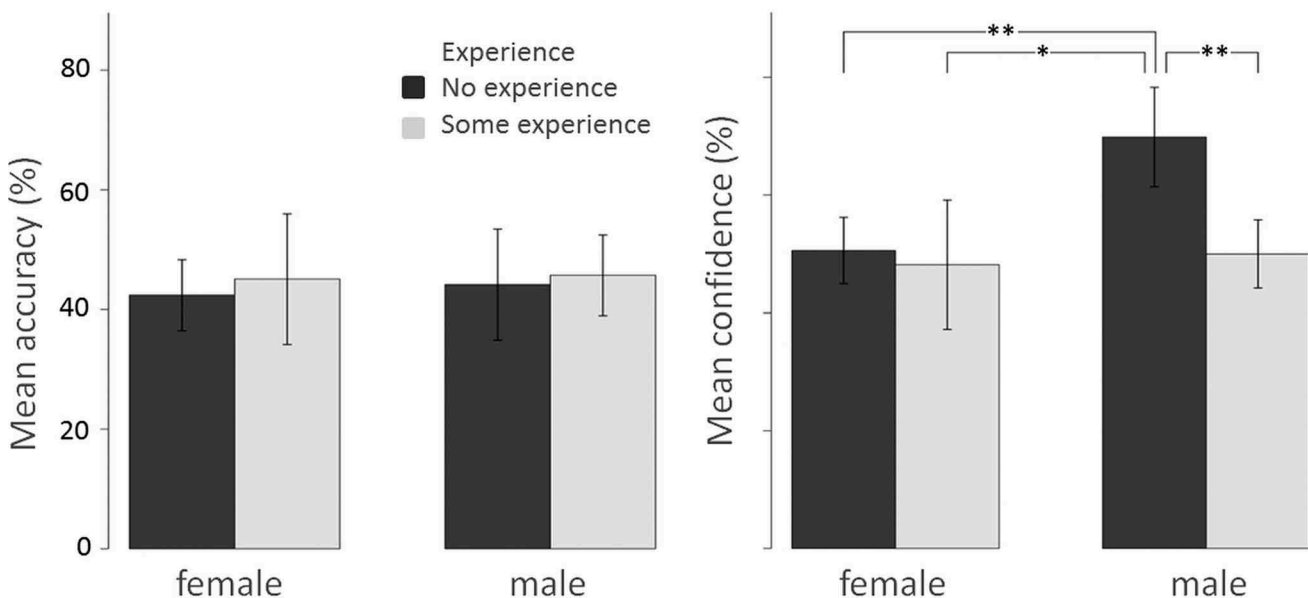


Figure 10. Gender and experience vs. accuracy (left) and confidence (right). Men with no experience display clearly higher levels of confidence than all other groups, even though they do not perform better ($* = p < .05$, $** = p < .01$). The error bars indicate 95% confidence intervals.

leads to the strongest relief inversion effect, and light from the “sides” leads to a nonnegligible ambiguity. Moreover, our analysis for the *best* light direction shed new light on the widely recommended cartographic convention of placing the light source at the 315°. While this convention is roughly in the right direction; our study demonstrated that incident light at 337.5° yields the highest accuracy rates among all investigated light directions, and the angles from 337.5° toward 0° may be better than the other way around. These observations confirm our hypotheses based on perceptual psychology literature: there is an *overhead illumination bias* (Kleffner and Ramachandran 1992) also in the case of SRMs, and accuracy is best when the light source is placed *above-left* (Mamassian and Goutcher 2001), though in our study, only *slightly* left.

While we believe our findings are robust, these results should be interpreted similarly as in any experiment because of the necessary limitations of controlled experiments. For example, each SRM was presented several times under a different illumination direction, and to prevent learning effects, we used an automatic randomization algorithm and shuffled the order. It is possible that our randomization algorithm placed the same terrain in succession once in a while, and thus did not *fully* solve the “learning effect” problem. Even though this possible effect should be distributed through randomization, i.e. all light directions will have it once in a while, if learning effect was not fully removed, it might explain why accuracy rates at northern angles never reach at full accuracy score at 100% (and southern angles do not hit the bottom at 0%). Of course there may be other experimental artifacts, but if, in some cases, participants have seen the same terrain in succession, they might have remembered the previous scene and this could affect their judgment; and thus may be reflected in the results. Another factor that is not fully controllable is the *landform direction* – the orientation of the feature we questioned (a valley or a ridge) in relation to the *light direction*. In other words, the terrain configuration might be an important factor. Therefore, for control purposes, we worked with eight different terrain configurations. These eight terrains are arguably a small sample for good external validity given the large variability between terrains. To counterbalance for this variability to some degree; we selected four *landform direction* categories. As mentioned earlier, incident light (roughly) parallel to the landform direction makes 3D shape perception difficult. This effect is qualitatively verifiable in individual terrains; and especially strong in some, e.g. in the terrain 50V_000 at 0°, where the feature is oriented exactly parallel to the incident light. Accordingly, the “dent” at

0° (respectively, the rise at 180°) in Figures 5 and 6 may have been caused (at least to some degree) by terrain 50V_000 in which the *landform direction* was oriented exactly parallel to the light direction at 0°. Note that the northern light angles are not particularly vulnerable for the relief inversion effect. If we did not have the terrain 50V_000 (oriented at 0°), the dent may not have been there; in other words, we would have an even stronger difference between the 0° and 315° in the McNemar test. As we controlled for this potentially biasing factor at the design stage (see Figure 3), we believe the effect is distributed; thus, overall, negligible for our main findings. Our results (as seen in Figures 5 and 6) are dependent on the characteristics of the studied terrains, as they are on the characteristics of the participants. Nonetheless, this is true for all experiments with visualizations to some degree. Since our overall findings are in agreement with previous studies (e.g. Bernabé Poveda and Çöltekin 2014) as well as the perceptual psychology studies, we believe the discussed limitations do not impose any notable harm to our findings.

More relief inversion than not: why might this be?

Our eight terrains had an equal number of valleys and ridges (four each); and the light direction was the only independent variable while other factors were kept as constant as possible. It is, therefore, reasonable to expect that success rates in identification of ridges and valleys are globally approximately even for the whole angular spectrum, i.e. half the time participants should experience relief inversion, while half the time they should not. Previously, in various perceptual psychology experiments, inversion rates were shown to be even for basic shapes (i.e. pits and holes, polo mint stimuli or similar) as the light direction was systematically rotated (e.g. Mamassian and Goutcher 2001; Gerardin, Kourtzi, and Mamassian 2010). In our experiment, globally, we observed that 53.3% of the responses suggest “inversed” perception, and 41.3% suggest “correct” perception while the remaining 5.4% were ambiguous. A closer look at the individual terrains further confirmed that only three of the terrains led to an approximately even ratio (50V_000, 20R_045, and 60V_045), while five of them indicated a very uneven ratio (Figure 11).

The fact that “correct” and “inverse” ratings are equal at the 278.2° and 68.5° (Figure 5) is a clear expression of this imbalance expressed in Figure 11. These two light directions are not at the mirrored opposite of each other by 180° on the angular spectrum as one would expect. In fact, for each of the eight terrains, 9 (or even more) out of 16 light directions yield more inverse than correct shape

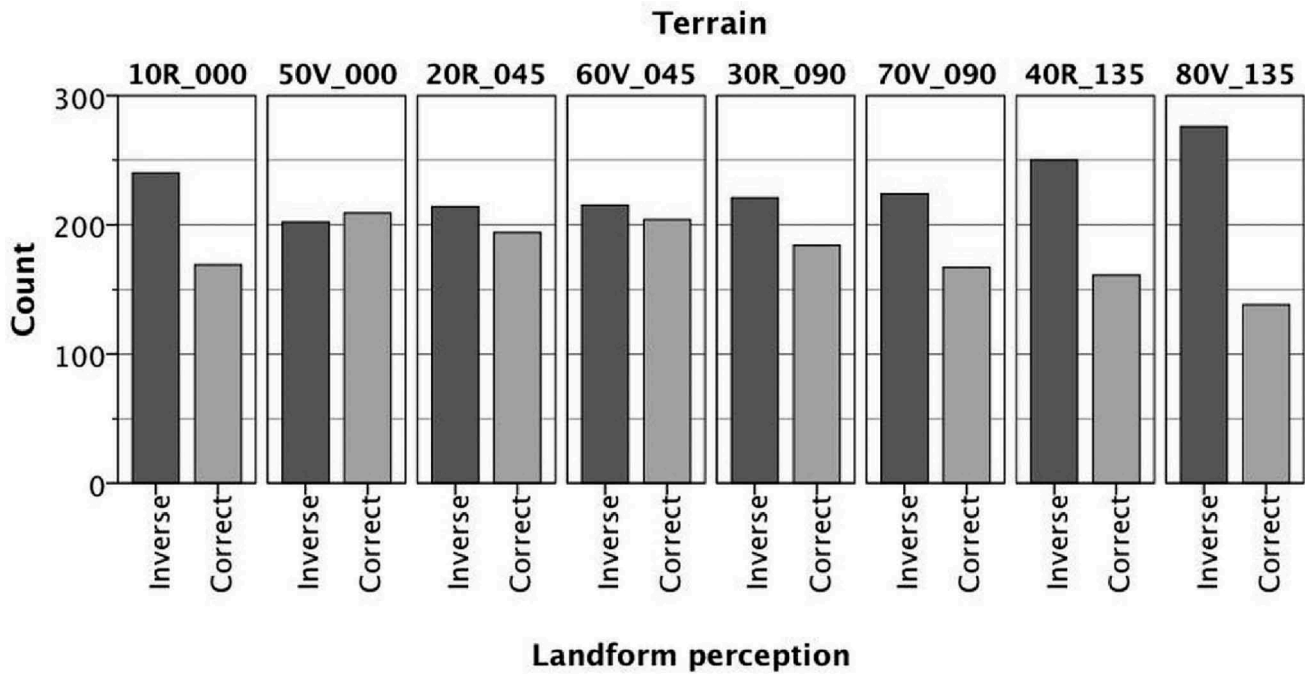


Figure 11. Absolute counts of responses suggesting correct vs. inverse perception per stimulus (terrain) aggregated over all 16 light directions. Left-to-right order is based on landform direction category.

perceptions. These differences can potentially be explained through the complex interplay between the HVS's assumed illumination position (i.e. from above-left), the incident light direction and the landform direction to some degree; but possibly also by the context provided by the terrain structures around the studied landform (the marked valley or ridge). While we observe and document this difference between the perceptual studies (no context, basic shapes) and ours (with context, a terrain), further experiments are necessary to pin down the exact combination of factors leading to this difference.

Confidence ratings in relation to light direction: are participants aware of the illusion?

One might expect that participants would feel less confident when the light shines laterally or from the southern directions, as terrains may look “odd” under these lighting conditions. As can be seen in Figure 6 however, the light direction seems to have no explicit effect on the confidence (all three confidence ratings *high*, *low*, *ambiguous* are rather horizontal). In other words, the participants appear to be largely unaware of the relief inversion effect. We believe that the slight variation in these three confidence ratings is mainly caused by specific terrain characteristics rather than by changes in the light direction: we observed in the analyses of individual terrains that the *ambiguous* and *low confidence* ratings considerably increase and the *high confidence* ratings decrease

when *landform direction* and light direction are parallel. When we plot the confidence ratings differently, however, another pattern emerges: Figure 12 shows a comparison of confidence ratings of opposite light directions in pairs: The northern light directions (337.5° , 0° , and 22.5°) have a higher confidence rate than their corresponding southern counterparts (157.5° , 180° , and 202.5°). Towards more laterally oriented pairs ($315^\circ/135^\circ$, $22.5^\circ/202.5^\circ$, $292.5^\circ/112.5^\circ$, and $67.5^\circ/247.5^\circ$), the confidence ratings are more similar. These observations indicate that northern light directions lead to slightly higher confidence than southern light directions.

We observed a left bias, but not a global convexity bias

Our observations confirmed the aforementioned *left bias* (Mamassian and Goutcher 2001) for 3D shape discrimination through shading in SRMs, as previously demonstrated in perceptual psychology: Western (i.e. *left*) light directions led to overall more correct responses than their corresponding eastern light directions in five out of seven angular pairs – with a statistically significant differences in four of the angular pairs. We observe a clear and consistent trend; out of the eight investigated terrains only one of them (80V_135) does not indicate the left bias, which can be explained by its landform direction category (135°). In our study, we observed the *left bias* at 13.4° . Sun and Perona (1998) observed the *left bias* at 23.3° for right-

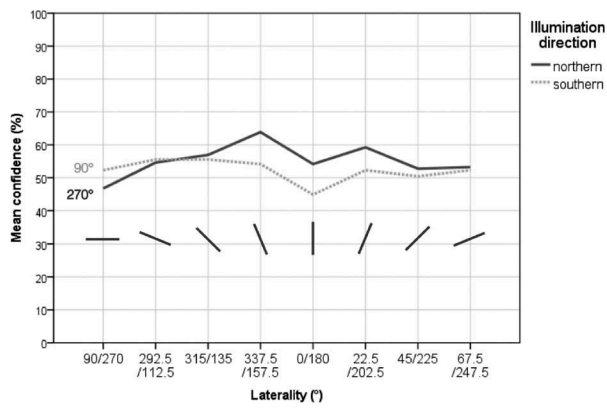


Figure 12. Confidence rates of opposite light directions with regard to laterality of the light direction. The gray bars below the graphs show the orientation of each light direction pair graphically. The mean confidence is calculated as the proportion of high confidence minus the proportion of ambiguous judgments.

handlers and at 7.9° for left-handers, Mamassian and Goutcher (2001) located it at 26° and Gerardin, Kourtzi, and Mamassian (2010) at 22° . While it appears somewhat weaker than the others, our results confirm the existence of the *left bias* also in geographic visualizations.

We observed no indication for a *global convexity bias* in our study. Convex forms were perceived marginally less often than concave forms (convex: 1625 times; concave: 1643 times). We believe these numbers do not suggest a bias either way. We also studied individual participants' number of convex/concave rating ratios and found a relatively large variation (maximum 1.61, and minimum 0.65). An exception was one participant who marked convex forms eight times more often than concave forms, however, this participant was one of the two participants excluded from the analysis (see *Participants* section), thus does not impact our results. If *global convexity bias* affected our results, one may also expect that, overall, the valley configurations would have lower accuracy rates than ridge configurations (because the *global convexity bias* would turn the ambiguous valley configurations into convex shapes). Conversely, one may expect a higher accuracy from the ridge configurations because of the *global convexity bias*. These expectations are not verifiable in our results: overall, there are 718 correct responses for valley (43.9%), and 708 correct responses for ridge configurations (43.4%). When individual light directions are compared, we observe that the mean of the four valleys and the mean of the four ridges also have very similar accuracy rates for most light directions. In cases where rates are somewhat different, it is probably caused by a certain terrain's landform direction (induced by its landform direction parallel to the light direction), rather than by the *global convexity bias*. We suspect, therefore, that any possible *global convexity bias* has largely been overwritten by the *overhead*

illumination bias in our experiment. It is possible that the *global convexity bias* plays a role under otherwise ambiguous lighting conditions. However, these hypotheses need to be tested with further experiments.

For whom is the illusion stronger? The impact of experience and gender

Besides the perceptual biases (*left bias* and *global convexity bias*), another factor that may have impacted our results is the participants' previous experience with the studied map type (SRM). Even though the aggregated analysis did not yield statistically significant differences in overall accuracy based on experience; descriptive statistics suggested that the participants with *some experience* using SRMs perform slightly better than participants with *no experience*, with the exception of northern-most light directions, where the results are reversed. This may suggest a pattern of overthinking, or subconscious overinterpreting by the participants with some experience, which in turn, leads to better performance when the terrain looks "odd," but leads them to make more mistakes too, when there is nothing to suspect. However, since we did not control for the individual differences as a part of our design, interpretation of these results should be taken with caution. Nonetheless, based on these initial observations, we speculate that experience may play a role, and our results warrant further testing.

While participants with some experience appeared to respond with arguably higher accuracy (based on descriptive statistics only), participants with no experience were more confident in their answers (statistically significantly). This difference in confidence is perhaps no surprise, as the more naïve participants might not pick up subtle cues regarding the geomorphology of the studied terrain, e.g. a river bed has a certain form. However, a deeper analysis including a gender breakdown suggested a strong overconfidence only by the male participants with no experience. The concept *male overconfidence* is a previously documented phenomenon in controlled lab studies (Dahlbom et al. 2011; Soll and Klayman 2004). In this study, we observed a nuanced version of male overconfidence, i.e. specifically demonstrated by the inexperienced male participants – even in a fairly straightforward perceptual cartographic task.

While the gender differences are not critical to our findings, experience (which experts should all have) is important because those who make maps are the experts. It is also important to remember that there may be nuanced group differences in the way we interpret visualizations, thus one should consider optimizing or personalizing the maps accordingly; and possibly be cautious

about the opinions of a group that might appear confident (i.e. confidence does not mean high performance).

Conclusions and outlook

Our comprehensive experiment contributes to the science and art of cartography through theoretical as well as applied results. First of all, our findings in this study lead to a clear recommendation in cartographic practice: we recommend cartographers to use 337.5° as the position of the (virtual) light source for illuminating SRMs, instead of the conventional 315°. 337.5° should be established as the new standard azimuth lighting direction in SRMs to avoid perceptual fallacies related to relief inversion effect. It is interesting to note that the 0° also leads to better results than 315°, thus the best window appears to be from 337.5° toward 0°. It is however also important to note that, while the light direction 337.5° offers the best results at the 96.3% accuracy rate (as well as leads to highest confidence rates, see Figures 5 and 6); participants perceive the terrains correctly in more than 80% of the cases within the 315°–45° window (Figure 6). Therefore, one might consider this window acceptable for placing the light source depending on the purpose of the visualization. Outside this window, accuracy rates start dropping steeply, and therefore, we discourage the use of light directions outside the boundaries 315°–45° for creating SRMs and strongly encourage the use of 337.5°. In cartography, a map designer can essentially arbitrarily decide the position of the light source (most software allow the entire 360 degrees), perhaps the SRM software could caution the map maker if they choose a light direction that is outside the safest 337.5°–0° window.

As a theoretical contribution, we observed that western (i.e. left) light directions yield predominantly higher accuracy rates than their corresponding eastern (i.e. right) light directions. Along with the theoretically “best” light direction of 13.4° to the left from the vertical, these results confirm that there is a *left bias* also with SRMs, therefore linking the perceptual psychology studies which are often conducted without context to cartographic experiments where the stimuli typically have a meaningful context. On the other hand, we did not observe a *global convexity bias*, which does not confirm the previous knowledge, and encourages new interdisciplinary research questions. Similarly, our examination of group differences hinted, and confirmed other studies, that *experience* in using a map type (in this case, SRMs) might influence even purely perceptual tasks (e.g. Brychtova and Coltekin 2014) such as identifying convex and concave shapes in an unlabeled, unfamiliar terrain; and warrants further research. Furthermore, our observations regarding confidence

differences linked to expertise and gender (overconfidence in inexperienced male participants) confirms the previous knowledge that we should be cautious in interpreting confidence as a sign of expertise; in fact it might mean the opposite in some cases, such as in our study.

The relief inversion effect (and its dependency on the light direction) has been known by cartographers for decades. Possibly mostly based on self-experimentation in traditional cartography; the effectiveness of NW lighting was proposed and widely adopted as a convention. It was not, however, empirically tested. Our study is the first comprehensive investigation that systemically assesses the impact of light direction on the “shape from shading” in a geographic context, and our findings enhance our understanding of a well-known and commonly practiced convention. In conclusion, with this study, we recommend the use of NNW lighting instead of NW for the most successful landform identification with SRMs. We plan to run further controlled studies to better understand how other cues (landcover, added depth cues) may impact the relief inversion effect and whether and how it can be avoided in all geographic visualization types.

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